15 Oscillations

Pages 306–310 Exam practice questions

1. The acceleration, energy and velocity in simple harmonic motion will all increase if the amplitude is larger. The frequency does not depend on the amplitude – answer C. [Total 1 mark]

2. The acceleration is a maximum at each end of the swing and the total energy and the frequency remain the same. The velocity is a maximum at the centre – answer D. [Total 1 mark]

3. Following on from question 2, at the end of the swing the pendulum momentarily comes to rest and so has zero velocity – answer D. [Total 1 mark]

4. \[ f = \frac{20 \text{ oscillations}}{16 \text{ s}} = 1.25 \text{ s}^{-1} \]
   \[ \omega = 2\pi f = 2\pi \times 1.25 \text{ s}^{-1} = 2.50\pi \text{ s}^{-1} \]
   The answer is D. [Total 1 mark]

5. The peak power is twice the average power – answer C [Total 1 mark]

6. a) A 12 V alternating current means that the peak voltage is \( \sqrt{2} \times 12 \text{ V} = 17 \text{ V} \). [1]
   A suitable setting for the input voltage might be 5 V per division as the trace would have an amplitude of ±3.4 divisions. [1]
   If the frequency is 50 Hz, the time period will be \( 1/50 \text{ s} = 20 \text{ ms} \). [1]
   A suitable time base setting might be 5 ms per division; then a wavelength would be 4 divisions. [1]
   b) Your sketch should be a sine wave that reflects your data. A sketch for the data above is shown in the figure below. [2]

7. a) The particular frequency at which a given part of a car vibrates is called its natural frequency of vibration. [1]
   b) Resonance occurs when an object is forced to vibrate and the frequency of the driving force is equal to the natural frequency of vibration of the object. This results in a maximum transfer of energy to the object, which vibrates with large amplitude. [2]
   c) Your graph should look like the figure below. [3]
d) This graph is also shown in the figure below. [2]

![Graph showing amplitude against frequency with resonant and applied frequency marked.]

e) If the suspension of a car is designed such that its natural frequency of vibration is nowhere near the typical engine frequency, and the suspension is also damped, the transfer of energy to the suspension will be minimised and unwanted resonance will be avoided. [2]

[Total 10 marks]

8 a) The relevant equation is \( a = -\omega^2 x \). As \( \omega \) is a constant, a graph of acceleration \( \omega \) against displacement \( x \) is a straight line through the origin of negative gradient \((-\omega^2\) – answer C). [1]

b) The potential energy has its maximum value at each end of the oscillation and is zero at the centre – think of a pendulum – so the answer is D. [1]

[Total 2 marks]

9 a) Your graphs should look like those in the figure below.

![Graphs showing velocity vs. time for oscillating and bouncing balls.]

b) For SHM, the acceleration must be proportional to the displacement and towards the equilibrium position. For the bouncing ball, as it descends, the gradient, and therefore the acceleration, is constant (if air resistance is ignored) and equal to the acceleration due to gravity, \( g \), while the ball is in the air. At the point of impact, this downward acceleration decreases and then suddenly becomes a large upward acceleration (very steep gradient) due to the contact force of the hard surface. Once the ball has left the surface, the acceleration is
15 Oscillations

Answers to Exam practice questions

once more $g$ downwards. The acceleration of the ball is therefore not proportional to the displacement and so the ball does not have SHM.

Note: The question also arises as to where the equilibrium position of the bouncing ball is – about which point does it oscillate? If this is taken to be the midpoint of its motion, then the acceleration is not always towards this point, again indicating that the motion is not SHM.

10 a) If the stroke (top to bottom) is 80 mm, the amplitude $A$ is 40 mm = $4.0 \times 10^{-2}$ m.

At 6000 rpm, the frequency $f = \frac{6000}{(60 \text{ s})} = 100 \text{ s}^{-1}$.

The SHM equation $x = A \cos(2\pi f)t$ is therefore

$$x = 4.0 \times 10^{-2} \cos(2\pi \times 100)t$$

$$= 4.0 \times 10^{-2} \cos 628t$$

b) i) From $a = -(2\pi f)^2 x$, maximum acceleration = $(2\pi f)^2 A$

$$\therefore a_{\text{max}} = (628 \text{ s}^{-1})^2 \times 4.0 \times 10^{-2} \text{ m}$$

$$= 1.6 \times 10^4 \text{ m s}^{-2}$$

ii) From $v = -(2\pi f)A \sin(2\pi f)$, maximum speed = $(2\pi f)A$

$$\therefore v_{\text{max}} = (628 \text{ s}^{-1}) \times 4.0 \times 10^{-2} \text{ m}$$

$$= 25 \text{ m s}^{-1}$$

c) Your graphs should look like those in the figure below

a) Displacement/mm

b) Velocity/ms$^{-1}$

Tip: To get full marks you must put values on the axes, if the values are known. To get the final mark, your velocity graph must be negative with respect to the displacement graph.
11 a) The amplitude, \( A \), is the distance the mass is pulled down from its equilibrium position, i.e. 60 mm. \[ \text{[1]} \]

b) \[ T = \frac{11.4 \text{ s}}{20} = 0.57 \text{ s} \] \[ \omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{0.57 \text{ s}} \] \[ = 11.02 \text{ s}^{-1} \approx 11 \text{ s}^{-1} \] \[ \text{[1]} \]

c) i) \[ a_{\text{max}} = -\omega^2 A = (11 \text{ s}^{-1})^2 \times 60 \times 10^{-3} \text{ m} \] \[ = -7.3 \text{ m s}^{-2} \] \[ \text{[1]} \]

ii) \[ v_{\text{max}} = \pm \omega A = \pm 11 \text{ s}^{-1} \times 60 \times 10^{-3} \text{ m} \] \[ = \pm 0.66 \text{ m s}^{-1} \] \[ \text{[1]} \]

d) Maximum KE = \( \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2} \times 0.400 \text{ kg} \times (\pm 0.66 \text{ m s}^{-1})^2 \) \[ = 0.087 \text{ J} \approx 0.09 \text{ J} \] \[ \text{[1]} \]

e) Maximum \( F = ma_{\text{max}} = 0.400 \text{ kg} \times 7.3 \text{ m s}^{-2} \] \[ = 2.9 \text{ N} = 3 \text{ N} \] \[ \text{[1]} \]

f) Spring constant (stiffness) is given by \( F = kx \).
\[ k = \frac{F}{x} = \frac{2.9 \text{ N}}{60 \times 10^{-3} \text{ m}} \] \[ = 48 \text{ N m}^{-1} \text{ (or 49 N m}^{-1} \text{ if all data is kept in the calculator)} \] \[ \text{[1]} \]

Tip: You might like to check for yourself that the equation \( T = 2\pi \sqrt{\frac{m}{k}} \) gives the same answer for \( k \). \[ \text{[1]} \]

g) Maximum \( E_p = \frac{1}{2}kA^2 = \frac{1}{2} \times 49 \text{ N m}^{-1} \times (60 \times 10^{-3} \text{ m})^2 = 0.087 \text{ J} \] This is equal to the maximum kinetic energy found in part (d), which it should be, because the total energy is conserved. \[ \text{[1]} \]

h) The mass is released at \( t = 0 \), at which point its KE is zero. As it passes through the midpoint of the oscillation for the first time (after \( \frac{1}{4}T \)) the mass reaches its maximum velocity and so has maximum KE of 0.09 J. When it reaches the top of the oscillation (after \( \frac{1}{2}T \)), it momentarily comes to rest, so its KE is zero. As it goes back down, its KE at the midpoint (after \( \frac{3}{4}T \)) is again 0.09 J, becoming zero as it reaches the bottom (after time \( T \)).

In between these values, the KE actually follows a \((\sin)^2\) curve as \( v = \pm \omega A \sin \omega t \), making the KE \( = \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t \). Because it is a \((\sin)^2\) curve it is positive for each half cycle and the curves are symmetrical about their midpoints. This is shown in blue in the figure below.

Likewise, \( E_p = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 A^2 \cos^2 \omega t \). This is plotted in red. Note that it is the mirror image of the KE curve.

The total energy is the sum of the KE and PE and, by conservation of energy, must be constant. This is therefore a horizontal straight line of value 0.09 J, as shown in purple. \[ \text{[Total 20 marks]} \]
a) For the motion of the particles to be simple harmonic, the particles must have an acceleration that is proportional to their displacement and which is directed towards their equilibrium position. [2]

b) As the total energy must – by the conservation of energy – remain constant, the total energy line must be a horizontal line at $2.0 \times 10^{-6}$ J. The kinetic energy must therefore be a mirror image of the potential energy curve so that $E_k + E_p = E_T = 2.0 \times 10^{-6}$ J throughout the motion. This is shown in the figure below. [2]

c) Drawing an analogy with a spring, the potential energy stored in the bonds is given by

$$E_p = \frac{1}{2}kx^2$$

From the graph, maximum $E_p = 2.0 \times 10^{-6}$ J when $x = 10 \times 10^{-2}$ m

$$k = \frac{2E_p}{x^2} = \frac{2 \times 2.0 \times 10^{-6} \text{ J}}{(10 \times 10^{-2} \text{ m})^2}$$

$$= 4.0 \times 10^{-4} \text{ N m}^{-1}$$
d) Buildings in earthquake zones can be designed to be safer by employing:

- ductile construction materials
- different types of damping mechanisms.

The ductile materials absorb the energy as they are compressed and then relax again (hysteresis), and the damping mechanisms reduce the energy transfer from the earthquake to the building. [2]

Tip: The question asks you to ‘explain’, so merely stating two ways (as in the bullet points) would only get half the marks.

13 a) At the midpoint, the student has no acceleration, so the resultant force on her must be zero. – the push of the seat must equal her weight. At the end of the oscillation, she will experience a resultant force towards the midpoint. This is provided by a component of the push of the swing. Your free-body diagram should therefore look like the figure below.

Tips: Remember that:
- free-body force diagrams should only show the forces acting on the body. If you put in any additional forces you will be penalised
- ‘gravity’ is not a force. You must state ‘weight’ or ‘mg’ to get the mark.

b) i) The motion may not be simple harmonic because:
- it is fairly heavily damped
- the girl may move her centre of gravity, and therefore change the resultant force, during the oscillation.
15 Oscillations

Answers to Exam practice questions

ii) The velocity at the midpoint of the first oscillation will be given by the gradient where the curve crosses the axis for the first time. If we draw a tangent to the curve at this point and extend it to the top and bottom edge of the grid we get:

\[ v = \frac{2.2 \text{ m}}{0.9 \text{ s}} = 2.4 \text{ m s}^{-1} \]

\[ [1] \]

\[ c) \] At the midpoint, the velocity has its maximum value when \( \sin (2\pi f) = 1 \)

\[ v_{\text{max}} = 2\pi f A = 2\pi \times 0.333 \text{ s}^{-1} \times 1.2 \text{ m} \]

\[ = 2.5 \text{ m s}^{-1} \]

\[ [1] \]

\[ d) \] If the girl on the swing were to be given a push each time she reached the end of each oscillation, the driving force (the push) would have the same frequency as her frequency of oscillation. She would therefore absorb this energy and oscillate with large amplitude – this is resonance.

\[ [3] \]

[Total 14 marks]

Stretch and challenge

14 a) For SHM, the body must have an acceleration that is proportional to its displacement from a fixed point and which is always directed back towards the fixed point.

b) The graph of acceleration as a function of displacement is shown in the figure below.

As the graph is a straight line through the origin it shows that the acceleration is proportional to the displacement. The negative slope indicates that the acceleration is always in the opposite direction to the displacement, i.e. back towards the centre of the oscillation.

c)

<table>
<thead>
<tr>
<th>From ( x = A \cos(2\pi f)t )</th>
<th>From ( x = A \sin(2\pi f)t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{dx}{dt} = -(2\pi f) A \sin(2\pi f)t )</td>
<td>( \frac{dx}{dt} = (2\pi f) A \cos(2\pi f)t )</td>
</tr>
<tr>
<td>( \frac{d^2x}{dt^2} = -(2\pi f)^2 A \cos(2\pi f)t )</td>
<td>( \frac{d^2x}{dt^2} = -(2\pi f)^2 A \sin(2\pi f)t )</td>
</tr>
<tr>
<td>( a = -(2\pi f)^2 x )</td>
<td>( a = -(2\pi f)^2 x )</td>
</tr>
</tbody>
</table>
The solution depends on the value taken for \(x\) when \(t = 0\). For the sine wave, \(x = 0\) when \(t = 0\), and for the cosine wave, \(x = A\) when \(t = 0\). There are an infinite number of solutions between these two as \(x\) can have any value between 0 and \(A\).

d) From \(x = A \cos(2\pi ft)\)
\[ v = \frac{dx}{dt} = -(2\pi f)A \sin(2\pi ft) \]
The maximum value that \(\sin(2\pi ft)\) can have is \(\pm 1\)
\[ v_{\text{max}} = \pm (2\pi f)A \]

[Total 16 marks]

15 a) For the mass to oscillate with SHM, the spring must obey Hooke’s law, i.e. its extension must be proportional to the resultant force acting on it.

b) For the spring:
\[ T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{0.400 \text{ kg}}{25 \text{ N m}^{-1}}} \]
\[ T = 0.795 \text{ s} \]
\[ f = \frac{1}{T} = \frac{1}{0.795 \text{ s}} = 1.26 \text{ Hz} \]

Maximum KE = \(\frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2} \times m \times (2\pi fA)^2 \)
\[ = \frac{1}{2} \times 0.400 \text{ kg} \times (2\pi \times 1.26 \text{ s}^{-1} \times 0.10 \text{ m})^2 \]
\[ = 0.125 \text{ J} \]

c) At the centre of an oscillation, the extension of the spring is just the extension produced by the mass of 0.400 kg.
\[ F = kx \quad \Rightarrow \quad x = \frac{F}{k} = \frac{0.400 \text{ N} \times 10 \text{ kg}^{-1}}{25 \text{ N m}^{-1}} \]
\[ = 0.16 \text{ m} \]

Elastic PE centre = \(\frac{1}{2}kx^2 = \frac{1}{2} \times 25 \text{ N m}^{-1} \times (0.16 \text{ m})^2 \)
\[ = 0.320 \text{ J} \]

At the bottom of the oscillation, the spring has been pulled down a further 10 cm, so total extension of spring = 0.16 m + 0.10 m = 0.26 m.
Elastic PE\text{bottom} = \frac{1}{2} \times 25 \text{ N m}^{-1} \times (0.26 \text{ m})^2 = 0.845 \text{ J}

At the top of the oscillation, the spring goes up 10 cm above the equilibrium position, so
extension of spring = 0.16 \text{ m} - 0.10 \text{ m} = 0.06 \text{ m}.

Elastic PE\text{top} = \frac{1}{2} \times 25 \text{ N m}^{-1} \times (0.06 \text{ m})^2 = 0.045 \text{ J}

\textbf{d)} At the bottom of an oscillation, the mass is 5 cm above the bench, so
GPE of mass at bottom = mg\Delta h = 0.400 \text{ kg} \times 10 \text{ N kg}^{-1} \times 0.05 \text{ m}
= 0.200 \text{ J}

At the top of an oscillation, the mass is 5 \text{ cm} + 10 \text{ cm} + 10 \text{ cm} = 25 \text{ cm} above bench, so
GPE of mass at top = 0.400 \text{ kg} \times 10 \text{ N kg}^{-1} \times 0.25 \text{ m}
= 1.000 \text{ J}

\textbf{e)} The graphs are shown in the figure below.

Points to note are:

- The gravitational potential energy of the mass will vary \textit{linearly} with the displacement (from $\Delta E = mg\Delta h$) – draw this first, from 0.200 J at the bottom to 1.00 J at the top. \[1\]
- The kinetic energy of the mass will be a maximum of 0.125 J at the midpoint and zero at the top and bottom of the oscillation, following an $x^2$ curve between these points. \[1\]
- Next plot the elastic potential energy at the bottom (0.845 J), middle (0.320 J) and top (0.045 J) and sketch a curve between the points. \[2\]
- At the top and bottom, when the kinetic energy of the mass is zero, the total energy of the system will be the sum of the potential energies (0.200 + 0.845 = 1.045 J at the bottom and 1.000 + 0.045 = 1.045 J at the top). They must, of course, be the same
because the total energy of the system is conserved. In other words the total energy is constant.